Abstract

Given a graph $G = (V, E)$ and a set of vertices $M \subseteq V$, with $|V| = n$, we say that $v \in V$ is *controlled* by M, if the majority of v neighbors (including itself) belongs to M. The set M defines a *monopoly* in G if M controls all vertices of V. In the *Monopoly Verification Problem* - MVP, we are given a set $M \subseteq V$ and two graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$, with $E_1 \subseteq E_2$. The objective is to find a sandwich graph $G = (V, E)$ (i.e., a graph where $E_1 \subseteq E \subseteq E_2$), such that M defines a monopoly in G. However, if the answer to the MVP is "NO", we have the *Max-Controlled Set Problem* - MCSP, whose objective is to find a sandwich graph $G = (V, E)$, such that the total number of controlled vertices is maximized. The MVP can be solved in polynomial time, the MCSP, however is NP-hard. In this work we describe the notion of *f-controlled* vertices and introduce the *Generalized Max-Controlled Set Problem* - GMCSP, where positive weights and gaps are associated to all vertices of V. In this case, the objective is to maximize the summation of the weights of all vertices f-controlled by M. We present a $\frac{1}{2}$-approximation algorithm for the GMCSP and a new procedure for finding feasible solutions based on the solutions of a linear programming relaxation. These solutions are then used in a local search procedure (Tabu Search with Path Relinking) looking for solutions of better quality. Finally, we present some computational results and we compare these results with the optimum solutions values obtained for small instances of the problem.

Key words: Sandwich Graph Problems, Approximation Algorithms, Construction Heuristics, Tabu Search, Path Relinking.